Mesurement of Water Density in the Fluids Laboratory of Inmetro

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Standards of Fluids Laboratory (Laflu)

Used as Standard in the Laboratory for measuring the density of liquids:

- Pycnometers (Reischauer and Gay Lussac);
- Hydrometers (L-20 and L-50);
- Digital Densimeter (Density Meter).

Pycnometers

Hydrometers

Density Meter

Measurement using the Pycnometer Method

- Measurement using Gravimetric Method;
- Range of Temperature: (15 - 40)^{o}C;
- Reference Liquid : bidistilled and deionized water;
- Standard : Reischauer Pycnometer.

Procedure of Measurement

Equation:

$$\rho_L(T_r) = \frac{M_L \cdot \left(1 - \frac{\rho_{ab}}{\rho_b}\right)}{V\left(1 + \gamma \cdot \left(T - T_r\right)\right)} + \beta_L \cdot \left(T_r - T\right) + \rho_a$$

where:

ρ_L : liquid density, in g/cm³;
M_L : liquid mass, in g;
V : volume of pycnometer, in cm³;
T : liquid temperature, in °C;
T_r : reference temperature, in °C;
γ : cubic expansion coefficient of glass , in ºC^{-1};
ρ_a: air density , in g / cm³;
ρ_{ab}: air density during the calibration of balance, in g / cm³;
ρ_b: density of standard weights , in g / cm³;
β_L: Bulk´s coefficient of Liquid, in ºC^{-1}.

Calculation of Uncertainty Measurement:

- **Input Quantities** :

$$\rho_L = F(M_L, V, T, \gamma, \rho_a, \rho_{ab}, \rho_b, \beta_L)$$

- Combined uncertainty:

$$u_C = \sqrt{(u_1)^2 + (u_2)^2 + (u_3)^2 + (u_4)^2 + (u_5)^2 + (u_6)^2 + (u_7)^2 + (u_8)^2 + (u_9)^2}$$

- Contribution uncertainty :

$$u_1 = C_1 \cdot u(M_L)$$

$$u_2 = C_2 \cdot u(V)$$

$$u_3 = C_3 \cdot u(T)$$

$$u_4 = C_4 \cdot u(\gamma)$$

$$u_5 = C_5 \cdot u(\rho_a)$$

$$u_6 = C_6 \cdot u(\rho_{ab})$$

$$u_7 = C_7 \cdot u(\rho_b)$$

$$u_8 = C_8 \cdot u(\beta_L)$$

$$u_9 = \frac{s}{\sqrt{n}}$$

Results

Input Quantity	Value of a Quanty	Best estimate	units	Distribution	sensitivity coefficient	Standard uncertainty	contribution of uncertainty	degrees of freedom
Xi	xi				ci	u(xi)	ui(y) ml	ν_i
M_L	100,89485	0,00062	g	normal	0,009881634	3,07957E-04	3,04312E-06	infinito
T_L	19,978	0,027	ºC	normal	0,00018144	1,35312E-02	2,45511E-06	infinito
V_p	101,1827	0,0012	cm^3	normal	-0,014588225	6,00000E-04	-8,75294E-06	428
ρ_a	0,00121	0,000002	g/cm^3	normal	1	1,16651E-06	1,16651E-06	infinito
ρ_b	8,00	0,10	g/cm^3	rectangular	2,76806E-05	5,77350E-02	1,59814E-06	infinito
ρ_{ab}	0,00120	0,000030	g/cm^3	normal	-0,184537184	1,50000E-05	-2,76806E-06	infinito
γ	0,000018	0,000008	1/ºC	rectangular	4,86172E-06	4,61880E-06	2,24553E-11	infinito
β_L	-0,000208	0,000115	$g/(cm^3.ºC)$	rectangular	0,021954545	6,62219E-05	1,45387E-06	infinito
$\delta\rho_L$ (aleatório)			g/cm^3	-	1	1,39508E-06	1,39508E-06	10

Temperature	Density of water	Expanded uncertainty	combined uncertainty standard	Covarage Factor (k)	effective degrees of freedom
ºC	g/cm^3	g/cm^3	g/cm^3		
20,00	0,99821	0,00002	0,000010	2,000	831

Observation

Equation of water density used in the determination of standard Volume is defined by ***Takenaka and Masui, in 1995,*** nowadays used by BIPM :

$$\rho_L(T) = a_5 \cdot \left[1 - \frac{(T + a_1)^2 \cdot (T + a_2)}{a_3 \cdot (T + a_4)} \right]$$

where:

- ρ_L**(T)** : water density temperature *(T)*, in the range of temperature between 0ºC a 40ºC and ambiental pressure of 101325 Pa, defined in $kg.m^{-3}$;
- **T** : water temperature in ºC ;
- a_1 = (-3,983035 ± 0,00067) ºC;
- a_2 = 301,797 ºC;
- a_3 = 522528,9 $ºC^2$;
- a_4 = 69,34881 ºC;
- a_5 = (999,974950 ± 0,00084) $kg.m^{-3}$.

References

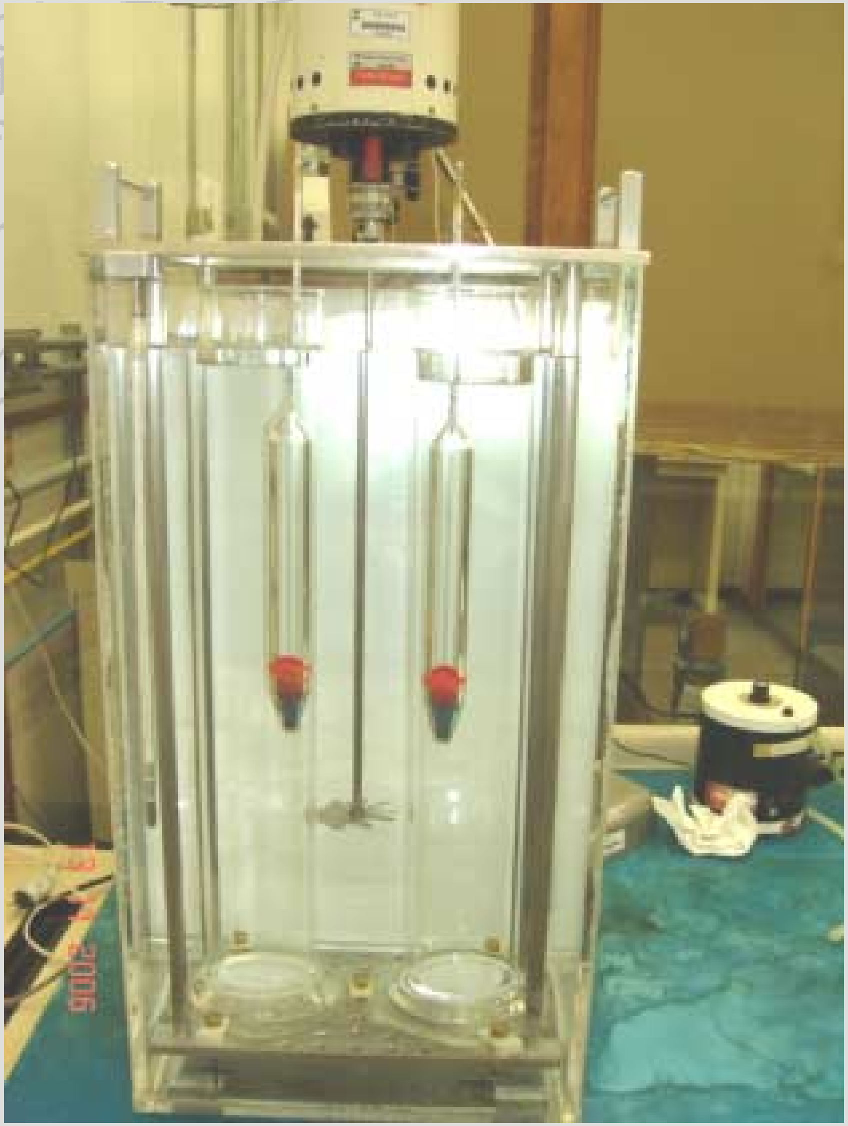
ISO 3838:1983 Crude Petroleum and Liquid or Solid Petroleum Products - Determination of Density - Capillary - Stoppered Pyknometer and Graduated Biacpillary Picknometer Methods.

ISO 3507:1999 Laboratory Glassware - Pyknometer

Measurement using the Hydrometer Method

- Standard : Hydrometer Type L-20 (traceable by PTB);
- Range of measurement : (0,980 - 1,000) g/mL;
- Temperature of measurement : 20ºC;
- Reference Liquid : bidistilled and deionized water.

Measurement System



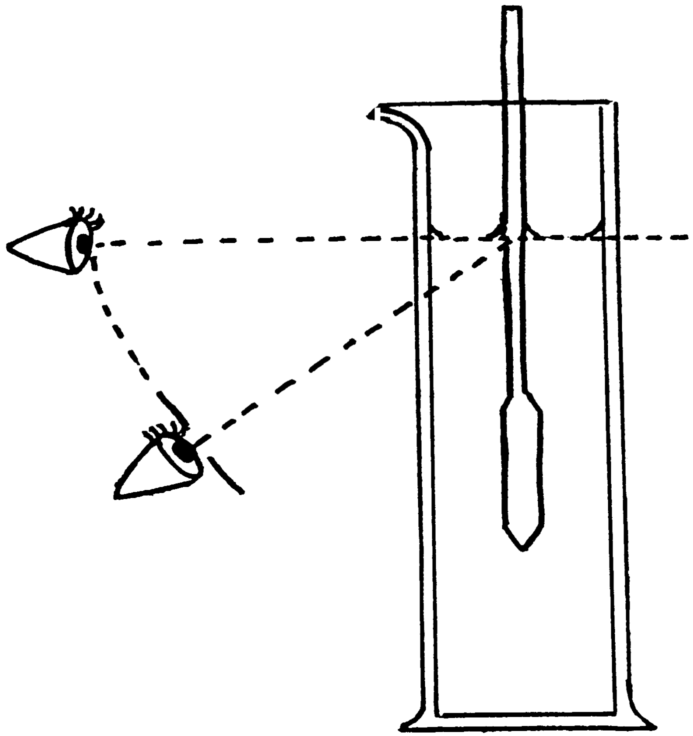
Reference Liquids

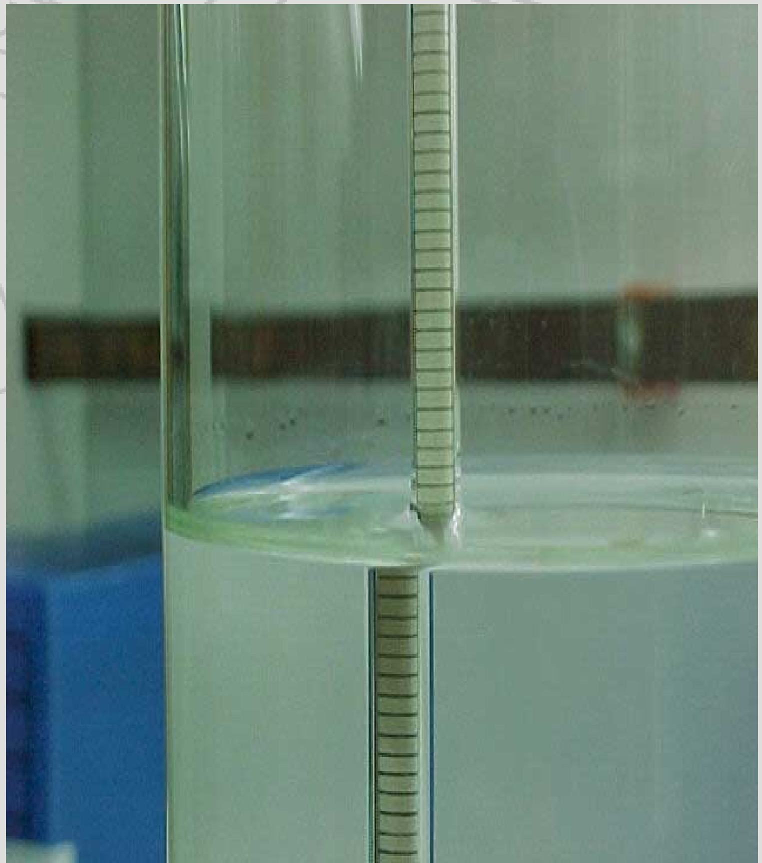
Reference solutions

Water Production System

Procedure of Measurement

- Reading liquid density in the horizontal level of the liquid surface





Equation :

$$\rho_L\left(T_r\right)=\rho_{ip}+C_{CT}+C_{\sigma}+C_T+C_M$$

where:

- ρ_{ip}: indication of standard hydrometer
- C_{CT}: Certificate of standard hydrometer Corretion
- C_{σ} : Surface Tension Corretion
- C_T: Temperature Corretion
- C_M: Meniscus Corretion

1- Surface Tension Corretion (C_σ) :

$$C_\sigma = \frac{0,41 \cdot (\sigma_L - \sigma_r)}{d_P \cdot \left(\frac{e_p}{r_p}\right) \cdot \rho_{ip}}$$

where:

$\boldsymbol{\sigma_L}$: surface tension of the liquid during measurement, in mN/m

$\boldsymbol{\sigma_L}$: surface tension reference of the liquid, in mN/m

$\mathbf{d_P}$: diameter of stem , in mm

$\mathbf{e_p}$: lenght between scales (of the hydrometer) , in mm

$\mathbf{r_p}$: nominal range of hydrometer , in mm

$\rho_{\mathbf{ip}}$: indication of standard hydrometer , in g/mL

Measurement of Surface Tension Liquid (σ_L)

- Standard : Tensiometer
- Method of measurement : Ring Method

Equation:

$$\sigma_L = F \cdot \sigma_a$$

where :

- σ_a : apparent surface tension (resulting of linearity of a tensiometer), em mN/m

$$\sigma_A = \frac{m_o \cdot g}{2\pi D}$$

m_o : mass of weight standard (shape of wire) used for determination of surface tension , in g

g : acceleration due gravity, in m/s²

D : diameter of ring, em cm

- **F** : Correction Factor of the surface tension (Using Harkins & Jordan equation)

$$F = 0{,}7250 + \left(\left(\frac{0{,}01452 \cdot \sigma_a}{C^2 \cdot (\rho_L - \rho_a)} \right) + 0{,}0453 - \frac{1{,}679 \cdot r}{R} \right)^{1/2}$$

C : mean circunference of ring , in cm.

R : radius of ring, in cm.

r : radius of wire , in cm.

σ_a : apparent surface tension , in mN/m

ρ_L : indication of standard hydrometer , in g/mL

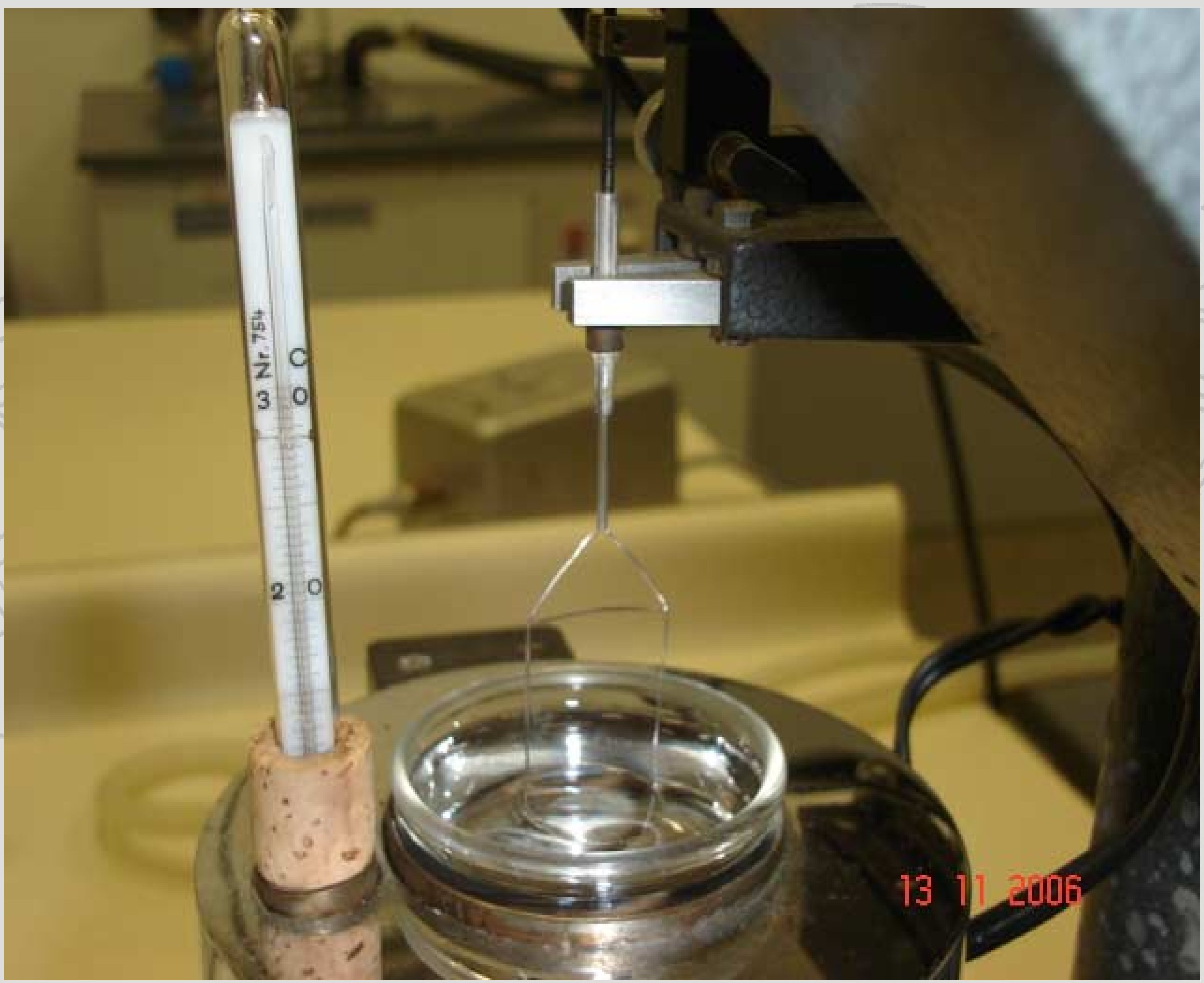
ρ_A : air density , em g/cm³

Tensiometer

Rings (Pt-Ir)

weight standard

Procedure of Measurement



2- Corretion Temperature (C_T)

$$C_T = \rho_{ip} . \gamma . (T_r - T)$$

where:

ρ_{ip}: indication of standard hydrometer , in g/mL

γ : cubic expansion coefficient of glass , in $^{o}C^{-1}$

T_r : reference temperature ,in ^{o}C

T : Temperature of the liquid during the measurement , in ^{o}C

3- Meniscus Corretion (C_M)

$$C_M = \frac{k.\sigma_L}{g.\left(\frac{e_P}{r_P}\right).d.\rho_{io}} \cdot \left(\sqrt{1 + \frac{2.g.d^2.\rho_{io}}{k \cdot \sigma_L}} - 1 \right)$$

Onde:

ρ_{io} : indication of standard hydrometer in the top of the meniscus , in g/mL

g : acceleration due to gravity , in m/s^2

σ_L : surface tension of the liquid , em mN/m

k=1 , indicated constant of standard hydrometer , in g/cm^3 or

k =1000 , indicated constant of standard hydrometer, in kg/m^3

e_p : lenght between scales (of the hydrometer) , in mm

r_p : nominal range of hydrometer , in mm

d : diameter of stem , in mm

Calculation of Uncertainty Measurement:

- Input Quantities :

$$\rho_L = F(\rho_{ip}, C_{CT}, T_L, \gamma, \sigma_L, \sigma_r, d_p, e_P, r_P)$$

- Combined uncertainty:

$$u_C = \sqrt{(u_1)^2 + (u_2)^2 + (u_3)^2 + (u_4)^2 + (u_5)^2 + (u_6)^2 + (u_7)^2 + (u_8)^2 + (u_9)^2 + (u_{10})^2}$$

- Contribution uncertainty :

$$u_1 = C_1 \cdot u(\rho_{ip})$$

$$u_2 = C_2 \cdot u(C_{CT})$$

$$u_3 = C_3 \cdot u(T_L)$$

$$u_4 = C_4 \cdot u(\gamma)$$

$$u_5 = C_5 \cdot u(\sigma_L)$$

$$u_6 = C_6 \cdot u(\sigma_r)$$

$$u_7 = C_7 \cdot u(d_p)$$

$$u_8 = C_8 \cdot u(e_p)$$

$$u_9 = C_9 \cdot u(r_p)$$

$$u_{10} = \frac{s}{\sqrt{n}}$$

Results

Standard :	L-20
Serial number:	189
Range of measurement:	(979,0 - 1000) kg/m³
Scale Value:	0,2 kg/m³
Reference Liquid:	ethanol water mixture

Input Quantity X_i	Value of a Quantity x_i	Best estimate	units	Distribution	sensitivity coefficient c_i	Standard uncertainty $u(x_i)$	contribution of uncertainty $u_i(y)$ ml	degrees of freedom ν_i
ρ_p	0,99790	0,000058	g/cm³	normal	9,9998E-01	2,88675E-05	2,88670E-05	infinito
C_{cp}	0,00025	0,000040	g/cm³	rectangular	1,0000E+00	2,30949E-05	2,30949E-05	infinito
σ_L	71,89	0,40	mN/m	normal	1,3106E-05	2,00000E-01	2,62113E-06	infinito
σ_r	70,55	1,00	mN/m	rectangular	-1,3106E-05	5,77350E-01	-7,56656E-06	infinito
d_p	5,70	0,10	mm	normal	-3,0810E-06	5,00000E-02	-1,54049E-07	infinito
e_p	1,10	0,10	mm	normal	-1,5965E-05	5,00000E-02	-7,98255E-07	infinito
r_p	0,0002	0,00005	g/cm³	rectangular	9,6589E-02	2,88675E-05	2,78828E-06	infinito
γ_p	0,0000250	0,000010	$°C^{-1}$	rectangular	-9,9790E-03	5,77350E-06	-5,76138E-08	infinito
T_L	20,010	0,019	°C	normal	1,9696E-04	9,57427E-03	1,88570E-06	infinito
$\delta\rho_L$			g/cm³	-	1	0,0000E+00	0,00000E+00	5

Temperature °C	scale indication g/cm³	Density of water g/cm³	Expanded uncertainty g/cm³	combined uncertainty standard g/cm³	Covarage Factor (k)	effective degrees of freedom
20,00	0,99790	0,99817	0,00008	0,000038	2,000	infinito

Standard :	L-20							
Serial number:	188							
ange of measurement:	(979,0 - 1000) kg/m^3							
Scale Value:	0,2 kg/m^3							
Reference Liquid:	mineral oil							
Input Quantity	Value of a Quantity	Best estimate	units	Distributior	sensitivity coefficient	Standard uncertainty	contribution of uncertainty	degrees of freedom
Xi	xi				ci	u(xi)	ui(y) ml	ν_i
ρ_{ip}	0,99787	0,0000612	g/cm^3	normal	9,9951E-01	3,06186E-05	3,0604E-05	infinito
C_{cp}	-0,00003	0,0000400	g/cm^3	rectangular	1,0000E+00	2,30940E-05	2,3094E-05	infinito
σ_L	71,89	0,40	mN/m	normal	1,3106E-05	2,00000E-01	2,62122E-06	infinito
σ_r	34,90	1,00	mN/m	rectangular	-1,3106E-05	5,77350E-01	-7,56682E-06	infinito
d_p	5,70	0,10	mm	normal	-8,5052E-05	5,00000E-02	-4,25259E-06	infinito
e_p	1,10	0,10	mm	normal	-4,4072E-04	5,00000E-02	-2,20361E-05	infinito
r_p	0,0002	0,00005	g/cm^3	rectangular	2,6664E+00	2,88675E-05	7,69715E-05	infinito
γ_p	0,0000250	0,0000100	ºC^{-1}	rectangular	-9,9787E-03	5,77350E-06	-5,76119E-08	infinito
T_L	20,010	0,01915	ºC	normal	1,9696E-04	9,57427E-03	1,88571E-06	infinito
$\delta\rho_L$			g/cm^3	-	1	1,17794E-05	1,17794E-05	5
Temperature	scale indication	Density of water	Expande ıncertaint	cined uncer standard	Covarage Factor (k)	ffective degrees of freedom		
ºC	g/cm^3	g/cm^3	g/cm^3	g/cm^3				
20,00	0,99787	0,99832	0,00012	0,000058	2,000	infinito		

Purpose Future

- Develop the Brazilian Method of Calibration of Hydrometer using Cuckow's Method;

- Develop the Method of Measurement of liquid Density using Hydrostatic Weighing

References

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